NAME ☞	
SCHOOL ☞	
TEACHER ☞	

Pre-Leaving Certificate Examination, 2016

Mathematics

Paper 1

Higher Level

Time: 2 hours, 30 minutes

300 marks

For examiner	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
Total	

School stamp

Running total	

Grade

Instructions

There are **two** sections in this examination paper:

Section A	Concepts and Skills	150 marks	6 questions
Section B	Contexts and Applications	150 marks	3 questions

Answer all nine questions.

Write your answers in the spaces provided in this booklet. You may lose marks if you do not do so. You may ask the superintendent for more paper. Label any extra work clearly with the question number and part.

The superintendent will give you a copy of the *Formulae and Tables* booklet. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

You will lose marks if all necessary work is not clearly shown.

You may lose marks if the appropriate units of measurement are not included, where relevant.

You may lose marks if your answers are not given in simplest form, where relevant.

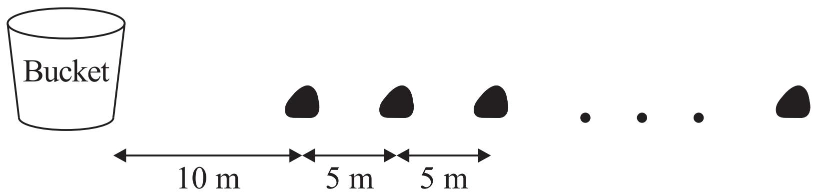
Write the make and model of your calculator(s) here:

Section A	Concepts and Skills	150 marks

Answer **all six** questions from this section.

Question 1 **(25 marks)**

(a) Fifty stones are placed in a straight line on level horizontal ground, at equal intervals of 5 m. A bucket is located on the same line 10 m away from the first stone. Tom, who can lift only one stone at a time, wishes to put all of them in the bucket. He starts from the bucket.



(i) Calculate the total distance, in km, that Tom travels to put all the stones in the bucket.

(ii) More stones are added and placed at the same intervals along the line. Find the number of extra stones added if the total distance that Tom travels to put all of the stones in the bucket is 27 km.

(b) Tom takes 5 seconds to retrieve the first stone and place it in the bucket. The time it takes him to place each subsequent stone in the bucket increases by 40%. How many stones can Tom retrieve and place in the bucket in 10 minutes?

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Question 2 **(25 marks)**

The functions f and g are defined for $x \in \mathbb{R}$ as

$$f : x \mapsto 1 - x \qquad \text{and}$$
$$g : x \mapsto 2x^2 - 9.$$

(a) Given that the function $h(x) = g \circ f(x)$, show that $h(x) = 2x^2 - 4x - 7$.

(b) **(i)** Express $h(x)$ in the form $a(x + b)^2 + c$, where a, b and c are constants.

(ii) Hence, or otherwise, find the co-ordinates of the turning point of the function $h(x)$.

(c) The function $k(x)$ is the image of $h(x)$ under a translation. The co-ordinates of the turning point of $k(x)$ are $(-1, -5)$. Find $k(x)$.

Question 3 **(25 marks)**

(a) Solve the inequality:

$$\frac{3x-1}{x+1} \leq \frac{5}{3}, \quad \text{where } x \in \mathbb{R} \text{ and } x \neq -1.$$

(b) **(i)** Solve the equation:

$$x + \frac{1}{x} = \frac{10}{3}, \quad \text{where } x \in \mathbb{R} \setminus \{0\}.$$

(ii) Use your answers above to solve the equation:

$$9^x + \frac{1}{9^x} = \frac{10}{3}, \quad \text{where } x \in \mathbb{R}.$$

(iii) Hence, or otherwise, solve the equation:

$$\log_3 x + \log_x 3 = \frac{10}{3}, \quad \text{where } x > 1.$$

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Question 4 **(25 marks)**

(a) $z = -\sqrt{3} + i$ is a complex number, where $i^2 = -1$.

(i) Write z in polar form.

(ii) Use De Moivre's theorem to find z^4, giving your answer in the form $a + bi$, where $a, b \in \mathbb{R}$.

(iii) Using the fact that z is one of the roots, or otherwise, find the other roots of z^4.

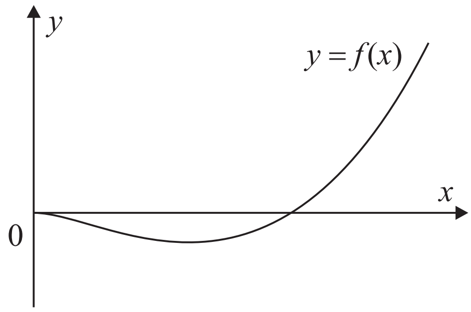
(b) Verify that $2i$ is a root of the equation $z^2 - (1 + 3i)z + (-2 + 2i) = 0$ and hence, find the other root of the equation.

Question 5 **(25 marks)**

The function f is defined as

$$f : x \mapsto x^2 \ln x, \quad \text{for } x \in \mathbb{R}, x > 0.$$

The diagram shows part of the graph of f.



(a) State whether or not f is injective.
Give a reason for your answer.

(b) Find $f'(x)$, the first derivative of $f(x)$.

(c) Find the co-ordinates of the local minimum point of the graph of f, in terms of e.

(d) Under what conditions is f bijective? For what domain and codomain does f meet this criteria?

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Question 6 **(25 marks)**

(a) Given a line segment of length one unit, show clearly how to construct a line segment of length $\sqrt{2}$ units, using only a compass and a straight edge. Hence, indicate a line segment of length $\sqrt{2} - 1$ units. Label each line segment clearly.

(b) $\sqrt{2} - 1$ is a root of $x^2 + bx + c = 0$, where $b, c \in \mathbb{Z}$.
Find the value of b and the value of c and hence, find the other root.

(c) **(i)** Explain what it means to say that $\sqrt{2}$ is not a rational number.

(ii) Use the method of proof by contradiction to prove that $\sqrt{2}$ is not a rational number.

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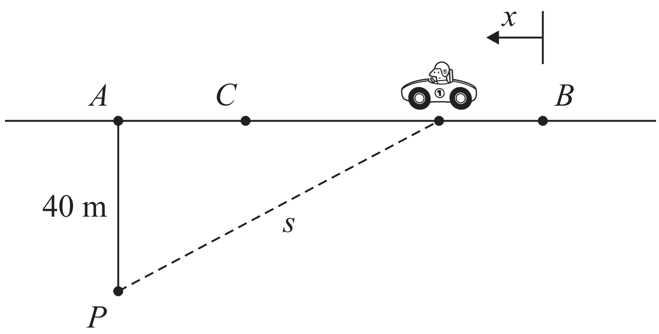
Section B	Contexts and Applications	150 marks

Answer **all three** questions from this section.

Question 7 **(50 marks)**

A speed camera is located at P, which is 40 m from the nearest point, A, on a straight level road. The camera monitors a car moving along the road from point B to A, which are 100 m apart. C is a point on the road which is 30 m from A.



Let s be the distance from the speed camera to the car after t seconds and let x be the distance the car has travelled from B at that instant. The speed of the car is given by the formula:

$$\frac{dx}{dt} = 25,$$

where speed is in metres per second. Both x and s are in metres.

(a) **(i)** Find the time it takes the car to reach C.

(ii) Express the distance of the car from A in terms of x and, hence, show that s, the distance between the car and the camera at time t, is $\sqrt{x^2 - 200x + 11{,}600}$.

(iii) Find $\frac{ds}{dx}$.

(iv) Hence, find the rate of change of s, with respect to t, when the car is at C.

(b) A second car passes B at the same time moving in the same direction towards A. The speed of the car is approximated by the following model:

$$v = 10e^{1\cdot 05t},$$

where v is the speed of the car in metres per second, and t is the time in seconds from the instant the car passes B.

(i) Use integration to find the time it takes this car to reach A. Give your answer correct to two decimal places.

(ii) Suggest **one** limitation of using an exponential function to model the speed of the car and give a possible reason for your answer.

Limitation:

Reason:

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Question 8 **(50 marks)**

On July 1st 2014, Meg borrowed €60 000 from her bank. She chose a payment-free option on the loan for 18 months. The bank charges interest that is equivalent to an annual percentage rate (APR) of 4·75%.

(a) Show that Meg owed the bank €64 325·37 on January 1st 2016.

(b) Meg agreed to repay the loan, plus interest, from January 2016 by a series of equal monthly payments over ten years, payable at the end of each month. By choosing the payment-free option on the initial period of the loan, Meg also agreed to pay a monthly interest rate of 0·425%, fixed for the term of the loan.

(i) Find, correct to five significant figures, the annual percentage rate (APR) that is equivalent to a monthly interest rate of 0·425%.

(ii) Calculate, correct to the nearest cent, the amount of each of Meg's monthly repayments.

(c) Meg's bank has agreed that, from January 1st 2022, to reduce the APR to 4·4% for the remainder of the loan if she makes all repayments up to that date on time and in full. The outstanding amount would be repaid in equal monthly repayments over the remaining term of the loan.

(i) By finding the present values of all her remaining repayments on January 1st 2022 and using the sum of a geometric series, calculate how much Meg still owes the bank on that date, correct to the nearest cent.

(ii) Calculate, correct to the nearest cent, the new amount that Meg will have to repay monthly for the remaining term of the loan.

(iii) Calculate the total amount of interest that Meg will pay over the entire term of the loan.

(iv) What advice would you offer Meg? Justify your answer by calculation.

Answer:

Justification:

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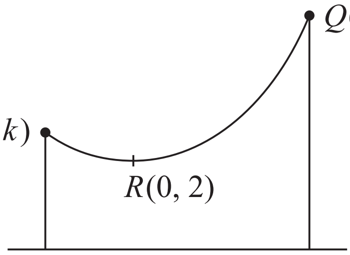
Question 9 **(50 marks)**

When a heavy chain hangs freely under the effect of gravity, and is supported only at each end, the shape that the chain forms is called a **catenary**, as shown.

Using a suitable choice of co-ordinates, the equation of a catenary can be written as:

$$y = \frac{a}{2}\left(e^{\frac{x}{a}} + e^{-\frac{x}{a}}\right), \text{ where } a \text{ is a constant.}$$



To demonstrate a catenary, a heavy chain is let hang freely between two points, P and Q, on opposite walls on level ground. The co-ordinates of P and Q are $(-2, k)$ and $(4, l)$ respectively, where k and l are constants.
Both x and y are in metres.

(a) **(i)** Given that the point $R(0, 2)$ lies on the path of the chain, show that the value of a is 2.

(ii) Find the height of the chain above ground level when the value of the x co-ordinate is 3, correct to one decimal place.

(iii) Find k, the value of the y co-ordinate of P, in terms of e.

(iv) Use your answer to part **(a)(iii)** above to show that l, the value of the y co-ordinate of Q, is $k^2 - 2$.

(b) **(i)** Show that R is the minimum point on the path of the chain.

(ii) Show, by calculation, that the slope of the tangent to the curve is always increasing from R to Q.

(c) Use integration to find the average height of the chain above ground level.
Give your answer in metres, correct to one decimal place.

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